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MONOTONICITY IN ELECTORAL SYSTEMS

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Much of the literature concerning the relative merits of alternative electoral rules is centered around the extent to which particular rules select "representative" legislatures. And an important concern in evaluating the "representativeness" of an electoral rule is whether or not the rule responds positively to changes in individuals' preferences, that is, whether or not the rule is monotonic. By explicitly considering electoral rules in the context of a complete electoral system—voting, selection of legislature, and legislative choice of policy—we argue that monotonicity in electoral systems is a nonissue: depending on the behavioral model governing individual decision making, either everything is monotonic or nothing is monotonic.

Broadly defined, a representative electoral system is a decentralized method for translating the preferences or interests of an electorate into policies via the selection of agents to a legislature. We can think of this process as the culmination of three distinct stages: (1) individuals make voting decisions based in some manner on their preferences; (2) the electoral rule in place determines who is elected to the legislature; and (3) the members of the legislature interact to derive a policy outcome. While outcomes from the second stage are determined mechanically, those from the first and third stages depend *inter alia* on strategic decisions of voters, candidates, and elected officials.

We have previously developed two alternative models of the legislative stage 3 and, in one case, examined the implications of the model for behavior at stage 1 (Austen-Smith and Banks 1990, 1988). Now, however, we suppress issues concerning behavior at the legislative stage and instead focus on stages 1 and 2, that is, on methods for translating votes into

legislatures and the associated behavior of the voters.

Much of the literature concerning the relative merits of alternative electoral rules is centered around the extent to which particular systems select "representative" legislatures. And an important concern in evaluating the representativeness of an electoral rule is whether or not the rule is monotonic. *Monotonicity* is a criterion capturing the idea that social choices should reflect positively changes in individuals' preferences: "It would be perverse in the extreme if increased votes for an alternative contributed to its defeat. Consequently, it seems an elementary requirement of sensible and fair choice that the decision rule respond positively, or at least non-negatively, to increase in individual evaluation of an alternative" (Riker 1982, 45). It is often argued, however, that many rules—in particular, "elimination" rules such as single transferable vote (STV) or runoff—violate monotonicity and thus are essentially undesirable (e.g., Brams and

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Fishburn 1984; Doron and Kronick 1977; Fishburn 1982; Riker 1982).

However, we argue that such a conclusion is unwarranted for two reasons. First, it is not the relationship between votes and legislatures that is of fundamental interest but rather the relationship between the underlying voter *preferences* and legislatures. The degree to which an electoral system is representative should not be judged with respect to voting decisions (i.e., processes) but rather with respect to actual preferences over outcomes (i.e., consequences). Analyzing this relationship between preferences and outcomes evidently requires a model of how voting behavior itself responds to changes in voters' preferences. Rather than specifying any particular model, however, we proceed by simply assuming the existence of some behavioral model of voting. We then show that if this model generates a unique prediction of how individuals vote for all possible profiles of underlying voter preferences, then for virtually all electoral rules the translation of voter preferences into legislatures via the behavioral model and the electoral rule will be nonmonotonic. For example, since "sincere" voting under most electoral rules gives a unique behavioral prediction, the resulting process will generate a nonmonotonic relationship between voter preferences and legislatures. On the other hand, the converse result holds if one admits behavioral predictions that are not necessarily unique for all profiles of underlying preferences. In particular, if one assumes the voters adopt Nash equilibrium strategies, then for *any* electoral rule the resulting translation of preferences into legislatures will be monotonic.

The second reason is that even if one has an intrinsic interest in process-oriented monotonicity, it turns out that such a requirement is not satisfied by *any* reasonable electoral rule. As remarked earlier, the literature contains several examples to illustrate the process-oriented

nonmonotonicity of particular electoral rules such as STV and runoff. This result implies that such examples can be constructed for virtually *all* electoral rules. Thus, our main conclusion is that monotonicity/nonmonotonicity in electoral systems is a nonissue.

Finally, it is worth emphasizing that although to our knowledge our arguments are novel, they are predicated on known results in the social choice literature concerning the existence of strategy-proof voting mechanisms.¹

The Model

Consider a political environment consisting of a set $N = \{1, \dots, n\}$ voters, in a single district (the extension to multiple districts is straightforward). In a legislative election, the voters elect ℓ representatives to the legislature. Let K denote the set of candidates competing for the district's seats; candidates may or may not be affiliated through some party structure. Thus $L = \{\lambda \subseteq K : |\lambda| = \ell\}$ is the set of all possible legislatures selected by the voters.

Voters determine a legislature through an *electoral mechanism* (V, γ) , where $V = \times V_i$ and V_i is voter i 's strategy set, N

and γ is a mapping $\gamma : V \rightarrow L$; we refer to γ as the *electoral rule*. Thus, given an electoral mechanism (V, γ) and a strategy profile $\mathbf{v} \in V$, the resultant legislature is given by $\gamma(\mathbf{v}) \in L$. Here we consider only the electoral rules for which the strategy sets of individuals $\{V_i\}$ can be equated with the set of orderings R_i . Thus for all i , $V_i = Q(L)$, the set of orderings of L .²

Once the legislature L has been determined, elected representatives select a policy outcome from some arbitrary feasible outcome space, X . Since our principal focus here is on the effects of various electoral laws, we suppress the behavioral model of the process by which legislatures

select outcomes by positing a reduced form expression $\sigma : L \rightarrow X$ summarizing the outcome of legislative decision making. Call σ a *legislative outcome function*. So given a strategy profile \mathbf{v} , voters determine policy outcomes in X through $\sigma(\gamma(\mathbf{v}))$.

So far we have described the second and third stages of an electoral system. To complete the picture, it remains to describe how individuals select their voting strategies. Rather than restrict attention at the outset to one behavioral model, we define an abstract behavioral rule as a mapping that assigns to every possible profile of voter preferences a profile of decisions. Formally, let $\beta : Q(L)^n \rightarrow Q(L)^n$ denote an arbitrary behavioral correspondence, where β may be functionally dependent on the electoral rule γ (e.g., the Nash equilibrium correspondence defined later).

Finally, we assume voter $i \in N$ has well-defined basic preferences \geq_i on X . Given these basic preferences and given the legislative outcome function σ , i has *induced preferences* R_i over the set of legislatures L given by, $\forall \lambda, \lambda' \in L$: $\lambda R_i \lambda'$ if and only if $\sigma(\lambda) \geq_i \sigma(\lambda')$. By definition, the structure of the domain of induced preferences over L depends on the structure of the domain of preferences over policy outcomes, X , and on the details of the legislative outcome function, σ . In particular, unrestricted domain on X does not necessarily imply unrestricted domain on L . Without specifying σ , however, nothing can be said about the structure of the domain of induced preferences. Given our focus, therefore, we make the conservative presumption that the domain of induced preferences over L is the unrestricted set of either weak or strong orderings, denoted $Q(L)$ and $Q^*(L)$, respectively. In what follows, we take the domain to consist of weak orders unless explicitly stated otherwise.

In sum, the formal representation of stages 1, 2, and 3 of an electoral system is

given by a specification of the triple (β, γ, σ) . The composition of these maps, $\beta \circ \gamma \circ \sigma$, thus characterizes the translation of voter preferences into policy outcomes through the electoral system.

As noted in the introduction, our principal aim is to establish results concerning monotonicity in electoral systems. To simplify the following arguments we distinguish two cases: single-member district ($\ell = 1$) and multimember district ($\ell > 1$). Consider these in turn.

Monotonicity in Single-Member Districts

By assumption, $\ell = 1$ so that $L = K$, and V_i is simply the set of orderings of candidates, K , competing for election. Thus, we can interpret a strategy by i , v_i , as a ranking of K ; so the statement " $x v_i y$ " means that x is ranked ahead of y according to i 's strategy v_i .

DEFINITION 1. Let $\alpha : Q(L)^n \rightarrow L$ be an arbitrary function from voter preferences into legislatures. We say that α is *monotonic* if $x = \alpha(\mathbf{v})$ and \mathbf{v}' is such that $x v_i y$ implies $x v'_i y$, for all $i \in N$ and for all $y \in L$, then $x = \alpha(\mathbf{v}')$.

Suppose we apply this definition to an electoral rule γ . Then monotonicity for single-member districts requires that if a candidate is elected under a strategy profile \mathbf{v} and if under a new strategy profile \mathbf{v}' every individual ranks that candidate at least as high as under \mathbf{v} , then the candidate should still be elected. Thus, monotonic electoral rules respond positively to changes in an individual's stated rankings of candidates. This description corresponds precisely to the notion of process-oriented monotonicity alluded to in the introduction.

Now consider the composition $\beta \circ \gamma$; observe that if β is single-valued, then, just as with an electoral rule γ per se, $\beta \circ \gamma$ is a mapping from preference profiles into

legislatures. Setting $\beta \circ \gamma = \alpha$ in definition 1 yields the requirement that if a candidate is elected under a preference profile R , and if under a new preference profile R' every individual ranks that candidate at least as high as under R , then the candidate should still be elected. And this is exactly the notion of consequence-oriented monotonicity.

The first result, Saijo's Theorem, makes use of the concept that a function $\alpha: Q(L)^n \rightarrow L$ is *constant* if for all $R, R' \in Q(L)^n$, $\alpha(R) = \alpha(R')$:

THEOREM 1. *If $\alpha: Q(L)^n \rightarrow L$ is monotonic, then α is constant (Saijo 1987).*

Hence, the requirement of monotonicity in an electoral rule is simply too stringent; the only electoral rule that satisfies process-oriented monotonicity is a trivial rule that selects the same legislature regardless of the individuals' voting decisions. Similarly, consequence-oriented monotonicity implies that the chosen legislature is never a function of the individuals' preferences if the behavioral rule $\beta(\cdot)$ is single-valued. For example, if $\beta(\cdot)$ is the identity function (i.e., voters behave "sincerely"), the translation of voter preferences into legislatures will be non-monotonic.

An important feature supporting Saijo's Theorem is the possibility of individual indifference over alternatives in L . For some electoral systems, however, this may not be appropriate; individuals may have only strict preferences over the alternatives they confront. So suppose we restrict attention to the set of linear orders over L , $Q^*(L)$. Say that a function $\alpha: Q^*(L)^n \rightarrow L$ is *dictatorial* if there exists an $i \in N$ such that for all profiles $P \in Q^*(L)^n$, $\alpha(P) P_i \lambda$ for all $\lambda \in L \setminus \alpha(P)$. Also say that α satisfies *citizen sovereignty* if for all $\lambda \in L$, there exists a profile $P \in Q^*(L)^n$ such that $\alpha(P) = \lambda$.

THEOREM 2. *Suppose $|L| \geq 3$. If $\alpha: Q^*(L)^n \rightarrow L$ is monotonic and satisfies*

citizen sovereignty, then α is dictatorial (Muller and Satterthwaite 1977).

Thus, as with theorem 1, requiring either process-oriented or consequence-oriented monotonicity (with $\beta(\cdot)$ single-valued) yields an essentially negative result.

A key element to both the results above is that the mapping α is single-valued. By definition, an electoral rule γ is necessarily single-valued; therefore, theorems 1 and 2 together imply that no reasonable electoral rule (i.e., nondictatorial and nonconstant) can satisfy process-oriented monotonicity. Similarly, if the behavioral model β is single-valued, the same conclusion applies to consequence-oriented monotonicity. However, many behavioral models are not necessarily single-valued, in which case the composite map $\beta \circ \gamma$ taking preference profiles into legislatures will not be single-valued and the theorems do not apply.³ Hereafter, we relax the assumption $\beta(\cdot)$ is single-valued and focus on a particular multivalued behavioral model, the Nash equilibrium correspondence.

DEFINITION 2. *A correspondence $\mu: Q(L)^n \rightarrow L$ is monotonic if $x \in \mu(R)$ and R' is such that $x R_i y$ implies $x R'_i y$, for all $i \in N$ and all $y \in L$, then $x \in \mu(R')$.*

Note that definition 1 above is a special case of definition 2 when μ is single-valued, that is, a function.

DEFINITION 3. *A strategy profile v^* is a Nash equilibrium if and only if for all $i \in N$ and all $v_i \in V_i$, $\gamma(v^*) R_i \gamma(v_i, (v_k^*)_{k \in N \setminus \{i\}})$.*

Let $\hat{\beta}(R; \gamma)$ denote the set of Nash equilibrium strategies given the electoral rule γ and true voter preferences $R = (R_i)_N$ over L .

In contrast to previous conclusions concerning monotonicity, the next result shows that given the behavioral model $\hat{\beta}(\cdot)$, all electoral rules are monotonic

when considering the relationship between true preferences and the chosen legislatures.

THEOREM 3. *For all electoral rules γ , the correspondence $\beta(\cdot; \gamma) \circ \gamma : Q(L)^n \rightarrow L$ is monotonic.*

Proof. Fix R and γ and let $\lambda \in L$ be supported as an equilibrium outcome by the strategy profile, v^* . Let R' satisfy the antecedent in the definition of a monotonic correspondence; the claim is that v^* is an equilibrium for R' . If not, there exists a v_i , $i \in N$, such that $\sim[\gamma(v^*) R'_i \gamma(v_i, (v_k^*)_{k \neq i})]$. But this implies $\sim[\gamma(v^*) R_i \gamma(v_i, (v_k^*)_{k \neq i})]$, a contradiction. QED

Hence, the Nash equilibrium correspondence generates consequence-oriented monotonicity for *all* electoral rules.

From theorem 3, it is immediate that results concerning the nonmonotonicity of STV and similar rules depend critically on the assumption that voters faithfully report their true preferences, independent of any strategic or other considerations. In particular, if γ is not monotonic, as is the case with nonconstant (theorem 1) or nondictatorial (theorem 2) electoral rules, then the following corollary shows that this assumption is inconsistent with optimizing behavior on the part of the voters:

COROLLARY 1. *If (V, γ) is such that γ is not monotonic, then there exists a preference profile $R = (R_1, \dots, R_n)$ such that $v_i = R_i$, for all i is not a Nash equilibrium (Muller and Satterthwaite 1977).*

Putting the results together yields our main conclusion for single-member districts: given that constant and dictatorial electoral rules are deemed unreasonable, either *no* reasonable electoral rule is monotonic or, relative to $\hat{\beta}(\cdot)$, *all* electoral rules are monotonic.

Monotonicity in Multimember Districts

By definition of a multimember district, $\ell > 1$. Consequently, although voters in N have well-defined induced preferences $(R_i)_N$ over L , such preferences are over *sets* of candidates of cardinality $\ell > 1$ rather than over individual candidates per se. Electoral rules such as STV, however, require voters to report rankings of individual candidates. Hence, $V_i = Q(K) \neq Q(L)$. Examples of the nonmonotonicity of, for example, STV, for multimember districts (e.g., Doron and Kronick 1977) use a notion of monotonicity related to that of definition 2 above. Specifically, if v and v' are lists of orderings of candidates in K and if they are such that for all voters i , $x v_i y$ implies $x v'_i y$, then the collection of candidates chosen under v coincides with that chosen under v' .

There are two difficulties with interpreting nonmonotonicity results based on this notion. The first is related to the distinction between process-oriented and consequence-oriented monotonicity. The second difficulty is that even allowing for a process-oriented judgment, this notion of monotonicity is inappropriate. The reason is that in multimember districts electoral rules do not, by definition, select single candidates but, rather, groups of candidates. Therefore, the relevant domain for any criterion such as monotonicity should be the set of individual preferences over such groups; these are, after all, the objects of choice. The mere fact that one candidate has risen in some individual's ranking does not necessarily tell us anything about changes in that individual's rankings over the relevant groups of candidates, that is, over L . Indeed, given our model, it is evident that voters do not even have well-defined preferences over candidates when district magnitude exceeds one. Basic preferences are over outcomes, X ; and these induce preferences

over multimember legislatures in L via the legislative outcome function, σ .

To illustrate the last point, suppose the district magnitude is 2 ($\ell = 2$), and suppose voter i ranks candidate a above candidate b . What does this imply about voter i 's preferences over electoral outcomes, given that the district elects not one candidate but two? We would claim that any implication consistent with this requires a restriction on voter i 's preferences over pairs of candidates; specifically, for any other candidate c , the pair of candidates (a, c) is preferred to the pair (b, c) .

In some circumstances the preceding restriction may be legitimate; yet it is a substantive restriction on individuals' preferences over electoral outcomes at the district level. For instance, in the double-member district example, let $K = \{a, b, c, d\}$ so that $L = \{(a, b), (a, c), (a, d), (b, c), (b, d), (c, d)\}$. Suppose the four candidates $\{a, b, c, d\}$ are associated with the points $\{1/8, 4/8, 5/8, 1\}$ respectively in the one-dimensional policy space $X = [0, 1]$ and suppose voter i has symmetric single-peaked preferences \geq_i on X with ideal point $1/2$. Finally, suppose the legislative outcome function is $\sigma(x, y) = [x + y]/2$, for all elected legislatures, $(x, y) \in L$. Thus, at the legislative stage, the final policy outcome is the compromise given by the midpoint of the elected candidates' positions. Then it is easy to check that the voter has induced preferences R_i over L such that (a, d) is strictly preferred to (b, d) , and (b, c) is strictly preferred to (a, c) . But this violates the necessary restriction on preferences over K ; the first ranking can only be rationalized at the individual candidate level by saying " i prefers a to b ," while the second ranking can only be rationalized by saying the converse. Thus, voter i is incapable of providing a ranking of individual candidates that reflects i 's preferences over electoral outcomes (i.e., pairs of candidates).

Notice that there exists a "natural" ordering of the candidates for voter i , given by their policy positions in X ; that is to say, $b >_i c >_i a >_i d$. Were the district magnitude equal to one, this ordering would certainly constitute i 's preferences over L . However, it is immaterial, given that the magnitude is two.

To avoid the difficulty with STV—or indeed with any multimember electoral rule where the strategy sets are ranking of individual candidates—one can simply require voters to report rankings of ℓ -tuples of candidates (i.e., rankings of L). But then all electoral rules are observationally equivalent to single-member electoral rules, where now "a member" is an element of L .

Given that this modification is implemented, theorems 1 and 2 apply: any reasonable electoral rule fails the process-oriented notion of monotonicity. Additionally, such rules fail consequence-oriented monotonicity when the behavioral rule $\hat{\beta}$ is single-valued. However, given β (i.e., the Nash equilibrium correspondence), the set of equilibrium outcomes will always respond positively to changes in voter preferences regardless of the electoral rule.

Conclusion

We have focused on the criteria of monotonicity and argued that as a normative critique the nonmonotonicity of electoral rules and systems has no bite. In particular, in any reasonable electoral system, outcomes will always respond positively to changes in voter preferences (given the Nash behavioral hypothesis) while not necessarily responding positively to changes in voters' reported preferences.

Finally, we emphasize that attempting to evaluate electoral systems by focusing exclusively on the electoral stages (stages 1 and 2) at the expense of the legislative

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stage (stage 3) is clearly inappropriate. Individuals have preferences over policy outcomes, and these induce preferences over the objects of choice (candidates or sets of candidates) through the legislative outcome function. The optimal voting decision for an individual in any election is therefore one that best promotes that individual's most preferred final outcome. Consequently, while an electoral rule may fail to satisfy a variety of appealing criteria with respect to recorded voting decisions, there is no reason to suppose the electoral system as a whole is not representative of individuals' policy preferences. Only through explicit consideration of all three stages can we have any confidence in judgments about any given electoral system's efficacy.

Notes

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1. This is in contrast to the literature on the existence of transitive, or acyclic, social choice rules.

2. Although most commonly employed electoral mechanisms do not require voters to report complete orderings, they can be modeled as if they do. For instance, plurality voting (with strict orderings) would simply ignore everything but the top-ranked alternative. Similarly, approval voting (with weak

orderings) would assign a value of one to every alternative in the top-ranked set and zero to all remaining alternatives.

3. It is important to note that if $\beta(\cdot)$ is set-valued, this does not imply that more than one candidate is elected to the legislature. Rather if $\mathbf{v}, \mathbf{v}' \in \beta(\cdot)$, this says that for some strategy profile \mathbf{v} , $\gamma(\mathbf{v})$ is selected, that for a distinct profile \mathbf{v}' , $\gamma(\mathbf{v}')$ is selected, and that both \mathbf{v} and \mathbf{v}' are consistent with the behavioral rule $\beta(\cdot)$.

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